

LECTURE NOTES

Course 6.041-6.431

M.I.T.

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Introduction to Probability

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Preface

These class notes are the currently used textbook for “Probabilistic Systems Analysis,” an introductory probability course at the Massachusetts Institute of Technology. The text of the notes is quite polished and complete, but the problems are less so.

The course is attended by a large number of undergraduate and graduate students with diverse backgrounds. Accordingly, we have tried to strike a balance between simplicity in exposition and sophistication in analytical reasoning. Some of the more mathematically rigorous analysis has been just sketched or intuitively explained in the text, so that complex proofs do not stand in the way of an otherwise simple exposition. At the same time, some of this analysis and the necessary mathematical results are developed (at the level of advanced calculus) in theoretical problems, which are included at the end of the corresponding chapter. The theoretical problems (marked by *) constitute an important component of the text, and ensure that the mathematically oriented reader will find here a smooth development without major gaps.

We give solutions to all the problems, aiming to enhance the utility of the notes for self-study. We have additional problems, suitable for homework assignment (with solutions), which we make available to instructors.

Our intent is to gradually improve and eventually publish the notes as a textbook, and your comments will be appreciated

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